

$$(2, -5, 3) = \alpha_1 (1, -3, 2) + \alpha_2 (2, -4, 1) + \alpha_3 (1, -5, 7)$$

$$(2, -5, 3) = (\alpha_1 + 2\alpha_2 + \alpha_3, -3\alpha_1 - 4\alpha_2 - 5\alpha_3, 2\alpha_1 + \alpha_2 + 7\alpha_3)$$

$$\alpha_1 + 2\alpha_2 + \alpha_3 = 2$$

$$-3\alpha_1 - 4\alpha_2 - 5\alpha_3 = -5$$

$$2\alpha_1 + \alpha_2 + 7\alpha_3 = 3$$

$$[A|b] = \begin{bmatrix} 1 & 2 & 1 & 2 \\ -3 & -4 & -5 & -5 \\ 2 & 1 & 7 & 3 \end{bmatrix} \approx \begin{bmatrix} 1 & 2 & 1 & 2 \\ 0 & 2 & -2 & 1 \\ 0 & -3 & 5 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} \alpha_1 & \alpha_2 & \alpha_3 & 2 \\ 1 & 2 & 1 & 2 \\ 0 & 2 & -2 & 1 \\ 0 & 0 & 4 & 1 \end{bmatrix} \quad \begin{bmatrix} 1 & 2 & 1 \\ 0 & 2 & -2 \\ 0 & 0 & 4 \end{bmatrix} \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix}$$

$A \quad X = B$

$$\alpha_3 = 1/4$$

$$2\alpha_2 - 2\alpha_3 = 1 \Rightarrow \alpha_2 = 3/4$$

$$\alpha_1 + 2\alpha_2 + \alpha_3 = 2 \Rightarrow \alpha_1 = 1/4$$

$$D = \{ (2, -5, 3), (1, -3, 2), (2, -4, 1), (1, -5, 7) \}$$

$$S_2 = \{ (1, 0), (0, 1) \}$$

$$S_3 = \{ (1, 0, 0), (0, 1, 0), (0, 0, 1) \}$$

$$5) \begin{bmatrix} 1 & 1 & 1 \\ 1 & 5 & 8 \\ 1 & 2 & 3 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 1 & 1 \\ 0 & 4 & 7 \\ 0 & 1 & 2 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 1 & 1 \\ 0 & 4 & 7 \\ 0 & 0 & 1 \end{bmatrix}$$

$$11) \text{span } V = \left\{ x \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} + y \begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix} + z \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \right\}$$

$$= \left\{ \begin{bmatrix} x & -y \\ -y & z \end{bmatrix}, x, y, z \in \mathbb{R} \right\}$$

$$12) V = \left\{ \begin{bmatrix} a & b & c \\ 0 & d & f \end{bmatrix}, a+b+c=0, a+d+f=0 \right\}$$

$$12) \quad V = \left\{ \begin{bmatrix} a & b & c \\ 0 & d & f \\ 0 & 0 & g \end{bmatrix} \right\}, \quad a+b+c=0, \quad a+d+g=0$$

$$\dim V = 4$$

rewriting

$$\begin{bmatrix} a & b & -a-b \\ 0 & d & f \\ 0 & 0 & -a-d \end{bmatrix}$$

$$\text{Basis} = \left\{ \begin{bmatrix} 1 & 0 & -1 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{bmatrix}, \begin{bmatrix} 0 & 1 & -1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \right\}$$

$$\left\{ \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix}, \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \right\}$$

$$\begin{aligned} & a, b, d, f \\ & (1, 0, 0, 0) \\ & (0, 1, 0, 0) \\ & (0, 0, 1, 0) \\ & (0, 0, 0, 1) \end{aligned}$$

$$13) \quad V = \{ (a, b, c) : b+c=0 \}$$

$$= \{ (a, b, -b) : a, b \in \mathbb{R} \}$$

$$\text{Basis} = \{ (1, 0, 0), (0, 1, -1) \}$$

$$\dim = 2$$

$$\begin{aligned} & a, b \\ & (1, 0) \\ & (0, 1) \end{aligned}$$

$$\text{Polynomial space: } P(\mathbb{R}) = \{ a_0 + a_1x + a_2x^2 + \dots + a_nx^n : a_0, a_1, a_2, \dots, a_n \in \mathbb{R} \}$$

$$P_n(\mathbb{R}) = \{ a_0 + a_1x + a_2x^2 + \dots + a_nx^n \}$$

$$\dim(P_n(\mathbb{R})) = n+1$$

$$\text{Basis} = \{ 1, x, x^2, \dots, x^n \}$$

$$10) \quad \alpha_1(x^2+1) + \alpha_2(3x-1) + \alpha_3(-4x+1) = 0$$

$$x^2(\alpha_1) + x(3\alpha_2 - 4\alpha_3) + (\alpha_1 - \alpha_2 + \alpha_3) = 0 + 0x + 0x^2$$

$$\alpha_1 = 0, \quad 3\alpha_2 - 4\alpha_3 = 0, \quad \alpha_1 - \alpha_2 + \alpha_3 = 0$$

$$\alpha_2 = \alpha_3 = 0 \quad \leftarrow \quad -\alpha_2 + \alpha_3 = 0$$

$$\begin{bmatrix} x^2 & x & 1 \\ 1 & 0 & 1 \\ 0 & 3 & -1 \\ 0 & -4 & 1 \end{bmatrix}$$